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# First-principles investigation of the structural, electronic and optical properties of olivine-Si<sub>3</sub>N<sub>4</sub> and olivine-Ge<sub>3</sub>N<sub>4</sub>

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#### Abstract

Si<sub>3</sub>N<sub>4</sub> and Ge<sub>3</sub>N<sub>4</sub> are important structural ceramics with many applications because of their outstanding high-temperature and oxidation-resistant properties. Two stable phases of them,  $\alpha$  and  $\beta$ , have thus far been synthesized. The highpressure and high-temperature spinel phases of these two materials were noticed to have wide, direct electronic band gaps that are comparable to those of the promising newly developed solid-state optoelectronic materials such as lasers and LEDs. Another high-pressure and high-temperature phase, the olivine phase, has also attracted attention recently. In the present work, the structural and electronic properties of the new olivine nitrides Si<sub>3</sub>N<sub>4</sub> and Ge<sub>3</sub>N<sub>4</sub> are studied by the FLAPW method with PBE-GGA exchange and correlation potential. The stability of the two materials and the transition pressure are investigated. It is found that olivine-Ge<sub>3</sub>N<sub>4</sub> is not stable and is difficult to be observed, while olivine-Si<sub>3</sub>N<sub>4</sub> can be synthesized under appropriate conditions. The atomic sites have been optimized and the ground-state properties such as equilibrium lattice constant, bulk modulus, band structure and density of states have been obtained. Furthermore, the dielectric function has been calculated based on the random phase approximation.

(Some figures in this article are in colour only in the electronic version)

#### 1. Introduction

Polymorphs formed from elements in group IV and group V make an interesting and important class of materials. Silicon nitride  $(Si_3N_4)$  is a well-known material because of its high fracture toughness, hardness and wear resistance [1, 2]. Another well-researched polymorph is  $Ge_3N_4$ .

The existence of several solid phases and the phase transitions among them have been attracting interest for years. Three phases,  $\alpha$  (P31c, hp28),  $\beta$  (P6<sub>3</sub>/m, hp14) and  $\gamma$  (spinel, Fd3m, cF56) phases of Si<sub>3</sub>N<sub>4</sub> and Ge<sub>3</sub>N<sub>4</sub> are well known from experiments [3–7]. Both the  $\alpha$  and  $\beta$ phases for Si<sub>3</sub>N<sub>4</sub> and Ge<sub>3</sub>N<sub>4</sub> are stable under ambient pressure, while the  $\beta$  phase is more stable than  $\alpha$  phase. The  $\alpha$  phase undergoes a transition into the  $\beta$  phase upon heating; however, the  $\beta \rightarrow \alpha$  phase transition has not yet been observed. The high-pressure phases of them are the  $\gamma$ phase.  $\gamma$ -Si<sub>3</sub>N<sub>4</sub> was first synthesized under a pressure of 15 GPa and at a high temperature of 2000 K [7]. It exhibits a wide direct band gap of 3.45 eV, though this is much smaller than those of  $\alpha$ - and  $\beta$ -Si<sub>3</sub>N<sub>4</sub> which have indirect band gaps [7–10]. The  $\beta \rightarrow \gamma$  phase transition of Ge<sub>3</sub>N<sub>4</sub> occurs near 12–15 GPa and above 1273 K [9].  $\beta$ -Si<sub>3</sub>N<sub>4</sub> undergoes a transition to the  $\gamma$  phase by a fast reconstructive process at a pressure above 20 GPa [11]. The  $\gamma$  phase is an isostructure of cubic- $C_3N_4$ , which is a new superhard substance predicted theoretically by Teter *et al* and whose bulk modulus is even larger than the value of diamond [12]. It has been shown that  $\gamma$ - $Si_3N_4$  and  $\gamma$ -Ge<sub>3</sub>N<sub>4</sub> have wide, direct electronic band gaps that are comparable to those of the promising newly developed solid-state optoelectronic materials such as GaN, InN, AlN [13], which have induced intensive researches on group VIA and VIB elements spinel nitrides from their structural, electronic properties to thermodynamic and mechanical properties [14-17].

A new phase in Ge<sub>3</sub>N<sub>4</sub> is a high-pressure phase with olivine-type structure (*Pnma*, oC28) predicted theoretically by Dong *et al.* It was indicated that a transition to a metastable olivine phase in Ge<sub>3</sub>N<sub>4</sub> could occur under appropriate pressure and temperature, if the  $\beta \rightarrow \gamma$  transition could be bypassed. They suggested that this could be achieved by exploiting the slow kinetics of the reconstructive transformation at low temperature [13]. Since in some isoelectronic oxides, such as Mg<sub>2</sub>SiO<sub>4</sub>, the olivine structure is the ground-state structure and it transforms into spinel at high pressure, it is interesting to explore further the olivine phase in nitrides. However, detailed information of the crystal structure, the stability and the condition of the phase transition of the olivine phase are not clear yet.

In present paper, we investigate the phase stability, and the structural, electronic and optical properties of both  $Si_3N_4$  and  $Ge_3N_4$  by *ab initio* calculations using the full-potential linearized augmented plane wave (FLAPW) method [18]. Our results show that the phase transition to olivine structure is rather difficult to occur in  $Ge_3N_4$  while it has a possibility of taking place in  $Si_3N_4$ .

The crystal structures of  $Si_3N_4$  and  $Ge_3N_4$  are described in section 2, which is followed by the calculation method and parameters in section 3. Results and discussion are given in section 4. Section 5 contains a summary and conclusion.

## 2. Crystal structure

Both Si<sub>3</sub>N<sub>4</sub> and Ge<sub>3</sub>N<sub>4</sub> have two stable structures at ambient conditions,  $\alpha$  and  $\beta$ . The highersymmetry  $\beta$  phase ( $P6_3/m$ , hp14) has a hexagonal lattice, while the lower-symmetry  $\alpha$  phase (P31c, hp28) is trigonal. Both structures consist of a tetrahedral network with each Si or Ge connected by three-coordinated N atoms; these have pyramid-like form in the  $\alpha$  phase, whereas the  $\beta$  phase can be regarded as stacking along the *c*-axis with a two-layer repeat pattern. In the spinel-structured ( $Fd\bar{3}m$ , cF56) polymorph  $\gamma$ -Si<sub>3</sub>N<sub>4</sub> and  $\gamma$ -Ge<sub>3</sub>N<sub>4</sub>, Si or Ge atoms are assumed to be in both tetrahedral and octahedral coordination to nitrogen. The olivine-named mineral is often used to designate members of the solid solution series bound by the end-members forsterite (Mg<sub>2</sub>SiO<sub>4</sub>) and fayalite (Fe<sub>2</sub>SiO<sub>4</sub>). The olivine structure type is commonly described as a somewhat distorted hexagonal close-packed array of anions in which one-eighth of the tetrahedral and one-half of the octahedral interstices are occupied by cations. This structure type can be designated as A<sub>2</sub>BX<sub>4</sub>, where A refers to octahedrally



Figure 1. The crystal structure of olivine-Si<sub>3</sub>N<sub>4</sub>.

Table 1. The Wyckoff parameters for each atomic species [19].

Atom	Wyckoff site	Coordinates
Si1	4a	(0, 0, 0), (1/2, 0, 1/2) (0, 1/2, 0), (1/2, 1/2, 1/2)
Si2, Si3 N4, N5	4c	$ \begin{array}{l} \pm(u, 1/4, v) \\ \pm(u+1/2, 1/4, 1/2 - v) \end{array} $
N6	8d	$ \begin{aligned} &\pm (r, s, t), \pm (r, 1/2 - s, t), \\ &\pm (r + 1/2, 1/2 - s, 1/2 - t), \\ &\pm (r + 1/2, s, 1/2 - t) \end{aligned} $

coordinated cations, B refers to tetrahedrally coordinated cations, and X refers to anions which are coordinated by three A cations and one B cation. In a unit cell of olivine- $Si_3N_4$  or olivine- $Ge_3N_4$ , A-type Si or Ge occupies the centres of distorted nitrogen octahedra, whereas B-type Si or Ge is tetrahedrally coordinated to nitrogen.

The olivine structure has space group Pnma, Pearson symbol oC28 with 8 A-type Si or Ge, 4 B-type Si or Ge and 16 N atoms which form 6 non-equivalent atom groups, Si1(Ge1), Si2(Ge2), Si3(Ge3), N4, N5 and N6. The Wyckoff sites and internal structural degrees of freedom are given in table 1, where the number following the element symbol of the atom is the identifier of the symmetrically non-equivalent atom group in the unit cell. These internal parameters have not been reported yet and need to be determined first in the calculations. The crystal structure of olivine-Si<sub>3</sub>N<sub>4</sub> is shown in figure 1, where Si1 and Si2 correspond to A, and Si3 corresponds to B.

#### 3. Calculation method and calculation parameters

In order to investigate the structural and electronic properties of olivine-Si<sub>3</sub>N<sub>4</sub> and Ge<sub>3</sub>N<sub>4</sub>, we performed calculation based on the local density approximation (LDA) with the generalized gradient approximation (GGA) [20, 21]. The calculations were performed using WIEN2k with a scalar relativistic effect [22]. The PBE-GGA was employed for the exchange and correlation potential [21]. The linearized augmented plane wave (LAPW) and augmented plane wave with local orbitals (APW + lo) methods were adopted as basis because the highest efficiency is found for a mixed basis set. The 'physically important' *l* quantum numbers are treated by APW+lo but the higher values of *l* were treated by the LAPW method [23]. Also for the

low-lying valence states, semi-core states, the local orbital (LO) was utilized. The basis set of Si, Ge and N for valence states used in the present calculations was

Si: 
$$(2p^6)3s^23p^2$$
,  
Ge:  $(3d^{10})4s^24p^2$ ,  
N:  $(2s^2)2p^3$ ;

here, the  $(2p^6)$  orbitals of Si,  $(3d^{10})$  orbitals of Ge and  $(2s^2)$  orbitals of N are treated as semicore states. For the potential and charge density representation, the maximum *l*-value for partial waves used inside atomic spheres was 10 and the one used in the computation of non-muffintin matrix elements was 4. In the interstitial region the basis functions were represented by Fourier series and the plane wave cut-offs of calculations were all set to be above 25 Ryd. The muffin-tin radii for Si, Ge and N were used (1.6, 1.7 and 1.5 au, respectively). The tetrahedron method was used for efficient sampling of the first Brillouin zone (BZ), and 126 points in the irreducible BZ [25], which corresponds to 1000 *k*-points in the whole BZ, were found to give a good convergence of the total energy and the ionic forces.

In order to obtain the internal structural parameters for  $Si_3N_4$  and  $Ge_3N_4$ , a reversecommunication trust-region quasi-Newton method from the Port library [26] was applied to optimize the equilibrium position of all individual atoms. Further optimization was performed with the constraint of maintaining the corresponding space-group symmetry.

## 4. Results and discussions

The main objective of present research was to investigate the unknown properties of newly predicted olivine-Si<sub>3</sub>N<sub>4</sub> and olivine-Ge<sub>3</sub>N<sub>4</sub>. However, in order to provide a consistent comparison of olivine phase with other phases which have already been reported, as well as a confirmation of accuracy of the present work, all calculations were been carried out for  $\alpha$ ,  $\beta$  and  $\gamma$  phases of both Si<sub>3</sub>N<sub>4</sub> and Ge<sub>3</sub>N<sub>4</sub> within the same theoretical framework. We present all results for the  $\gamma$  phase together with the olivine phase, but only include the  $\beta$  phase results in the discussion of phase stability and energy gaps.

#### 4.1. Structural optimization and stability

So far olivine-Si<sub>3</sub>N<sub>4</sub> and olivine-Ge<sub>3</sub>N<sub>4</sub> have not been researched sufficiently and no structural parameters have been published. It was necessary to optimize the lattice parameters first. There are several steps for determining the equilibrium lattice parameters. First the ideal atomic positions of olivine phase material shown in table 2 were used with the lattice parameters from figure 3 in the paper by Dong *et al* [13]. Then the atomic positions were relaxed globally using the force-minimization technique. Afterwards, the total energies were calculated by fixing both the optimized atomic positions and c/a ratio with different compressed and expanded volumes. Finally, by fitting the total energies versus volume to Murnaghan's equation of state [28], we derived the equilibrium properties such as lattice parameter, the bulk modulus *B*, and the pressure derivative of the bulk modulus B' from the equation of state.

The calculated internal coordinates for olivine-phase and spinel-phase  $Si_3N_4$  and  $Ge_3N_4$ are listed in table 2. It can been seen that for the  $\gamma$  phase, our results are very close to the measurements and other calculations, which confirms the reliability of our approach. For the olivine phase, the results show a small deviation of the optimized structures from the ideal ones. These calculations provide detailed information on the internal parameters for both olivine- $Si_3N_4$  and olivine- $Ge_3N_4$  for the first time.

	Olivine phase						γ phase
	Atom	и	υ	r	S	t	w
Ideal							
	Si2 or Ge2	0.25	0				0.3750
	Si3 or Ge3	0.0833	0.375				
	N4	0.0833	0.75				
	N5	0.4167	0.25				
	N6			0.1667	0	0.2500	
Optimized							
Si <sub>3</sub> N <sub>4</sub>	Si2	0.2900	0.9662				0.3838
	Si3	0.0918	0.4398				
	N4	0.0951	0.8127				
	N5	0.4376	0.2505				
	N6			0.1527	0.0011	0.2736	
Experiment							0.3875 <sup>a</sup>
*							0.3833 <sup>b</sup>
Other calculations							0.3843 <sup>c</sup>
							0.3844 <sup>d</sup>
Ge <sub>3</sub> N <sub>4</sub>	Ge2	0.2800	0.9887				0.3833
5	Ge3	0.0945	0.4476				
	N4	0.0940	0.8210				
	N5	0.4372	0.2448				
	N6			0.1611	0.0052	0.2620	
Experiment							0.3827 <sup>e</sup>
Other calculations							$0.3830^{f}$
							0.3841 <sup>d</sup>

**Table 2.** Optimized internal parameters in  $\gamma$ -, olivine-Si<sub>3</sub>N<sub>4</sub> and  $\gamma$ -, olivine-Ge<sub>3</sub>N<sub>4</sub> in unit cell.

<sup>a</sup> Reference [4]; <sup>b</sup> Reference [24]; <sup>c</sup> Reference [10]; <sup>d</sup> Reference [27]; <sup>e</sup> Reference [5]; <sup>f</sup> Reference [13].

The results of calculated equilibrium properties of the four systems are summarized in table 3, together with available experimental data and other calculations. For the equilibrium lattice constants, one sees that our results for the  $\gamma$  phase (14.7526 au for Si<sub>3</sub>N<sub>4</sub>, 15.7137 au for Ge<sub>3</sub>N<sub>4</sub>) achieve an excellent agreement with the experiments (14.7399 au for Si<sub>3</sub>N<sub>4</sub>, 15.6847 au for Ge<sub>3</sub>N<sub>4</sub>) [4, 6], better than all other calculations. Within same theoretical framework, the present calculation obtained the equilibrium lattice constants of olivine phase as *a* = 18.7717, *b* = 10.5769, *c* = 8.6712 au for Si<sub>3</sub>N<sub>4</sub>, and *a* = 19.7117, *b* = 11.1474, *c* = 9.1019 au for Ge<sub>3</sub>N<sub>4</sub>, respectively.

Shown in figure 2 are total energy versus volume curves for the  $\beta$ ,  $\gamma$  and olivine phases of Si<sub>3</sub>N<sub>4</sub> and Ge<sub>3</sub>N<sub>4</sub>. The solid line represents the  $\beta$  phase, the dashed line the  $\gamma$  phase, and the dotted line the olivine phase. For both Si<sub>3</sub>N<sub>4</sub> and Ge<sub>3</sub>N<sub>4</sub>, the results show that the  $\beta$  phase is most stable at the ground state; it is possible to transform it into the  $\gamma$  phase under a high pressure, which coincides with the experimental observations. Another high-pressure phase, olivine, lies higher in energy than the  $\beta$  and  $\gamma$  phases. It is known that in isoelectronic oxides, such as Mg<sub>2</sub>SiO<sub>4</sub>, the olivine structure is the ground-state structure and it transforms into spinel at high pressure; this is obviously different in the case of nitrides.

In figure 2(a), Si<sub>3</sub>N<sub>4</sub>, according to our calculation the energy difference between  $\beta$ -Si<sub>3</sub>N<sub>4</sub> and  $\gamma$ -Si<sub>3</sub>N<sub>4</sub> is 146 meV/atom. By drawing the common tangent line of total energy versus



Figure 2. The calculated total energy versus volume curves of (a) Si<sub>3</sub>N<sub>4</sub> and (b) Ge<sub>3</sub>N<sub>4</sub>. The solid line represents the  $\beta$  phase, the dashed line the  $\gamma$  phase, and the dotted line the olivine phase. The parameters are provided in table 3.

Table 3. Equilibrium lattice parameters, band gap and bulk modulus.

	Olivine-Si <sub>3</sub> N <sub>4</sub>	$\gamma$ -Si <sub>3</sub> N <sub>4</sub>	$\beta$ -Si <sub>3</sub> N <sub>4</sub>	Olivine-Ge <sub>3</sub> N <sub>4</sub>	γ-Ge <sub>3</sub> N <sub>4</sub>	β-Ge <sub>3</sub> N <sub>4</sub>
Lattice constant						
<i>a</i> (au)	18.7717	14.7526		19.7117	15.7137	
b (au)	10.5769			11.1474		
<i>c</i> (au)	8.6712			9.1019		
Experiment		14.7399 <sup>a</sup>			15.6847 <sup>b</sup>	
		14.6340 <sup>d</sup>			15.5209 <sup>c</sup>	
		14.6149 <sup>e</sup>				
Others' calculation		14.8102 <sup>g</sup>			15.5165 <sup>g</sup>	
		14.8092 <sup>h</sup>			15.4345 <sup>i</sup>	
Band gap (eV)						
	2.99	3.21	4.29	1.55	1.90	2.17
Experiment			4.6-5.3 <sup>j</sup>			$\sim \!\! 4.5^k$
Other calculations		3.45 <sup>g</sup>	4.26 <sup>i</sup>		2.22 <sup>g</sup>	2.45 <sup>i</sup>
		3.45 <sup>h</sup>			2.17 <sup>i</sup>	
B (GPa)						
	262.9	292.3		182.3	216.3	
Experiment		$317\pm11^{\rm f}$			296 <sup>c</sup>	
Other calculations		280.1 <sup>g</sup>			268.6 <sup>g</sup>	
		280 <sup>h</sup>			240 <sup>i</sup>	
Β′						
	3.93	4.35		4.61	4.61	
Experiment		$2.3\pm2.1^{\rm f}$			4.0 <sup>c</sup>	
Other calculations		3.76 <sup>g</sup>			3.14 <sup>g</sup>	
		3.48 <sup>h</sup>			4.5 <sup>i</sup>	

<sup>a</sup> Reference [4]; <sup>b</sup> Reference [6]; <sup>c</sup> Reference [5]; <sup>d</sup> Reference [21]; <sup>e</sup> Reference [24]; <sup>f</sup> Reference [25]; <sup>g</sup> Reference [27]; <sup>h</sup> Reference [10]; <sup>i</sup> Reference [13]; <sup>j</sup> Reference [30]; <sup>k</sup> Reference [31].

volume curves of  $\beta$  and  $\gamma$  phases which indicates the equilibria of these two phase, the  $\beta - \gamma$ transition pressure can be estimated from the slope as 7.1 GPa. This implies that the transition from  $\beta$  to  $\gamma$  phase could be easily observed. Looking at olivine-Si<sub>3</sub>N<sub>4</sub>, we find that it is also

possible to draw a common tangent line of the two energy curves of olivine- and  $\beta$ -Si<sub>3</sub>N<sub>4</sub>; the transition pressure estimated from the slope is 24.8 GPa. This predicts that when Si<sub>3</sub>N<sub>4</sub> is compressed under high pressure from the equilibrium state of the  $\beta$  phase, the phase transition from  $\beta$ -Si<sub>3</sub>N<sub>4</sub> to olivine-Si<sub>3</sub>N<sub>4</sub> can occur at this pressure if the  $\beta \rightarrow \gamma$  transition could be bypassed, and that olivine-Si<sub>3</sub>N<sub>4</sub> might exist as a metastable phase. Noticing there is a rather large energy difference of 345 meV/atom between olivine-Si<sub>3</sub>N<sub>4</sub> and  $\beta$ -Si<sub>3</sub>N<sub>4</sub>, such a phase transition into a high-energy state at high pressure generally requires a large amount of internal energy to overcome the energy barrier [13]; the high temperature would be necessary to exert an additional contribution to internal energy from the entropy contribution. In fact, the discussion of such a kind of transformation with a temperature effect should consider the free energy instead of the internal energy at the ground state. However, the internal energy still gives important information about the stability, as shown in current systems.

In figure 2(b), Ge<sub>3</sub>N<sub>4</sub>, an energy difference of 55 meV/atom is obtained between  $\beta$ - $Ge_3N_4$  and  $\gamma$ - $Ge_3N_4$ , which agrees with the result obtained by Dong *et al* [13]. The calculated transition pressure between the phases is about 13.7 GPa, which is also in a good agreement with the experiment [9]. However, it is noticed that the bottom point of the energy of olivine-Ge<sub>3</sub>N<sub>4</sub> is fully enclosed inside both  $\beta$ - and  $\gamma$ -Ge<sub>3</sub>N<sub>4</sub>, so that it is impossible to have a common tangent line between either  $\beta$ - and olivine-Ge<sub>3</sub>N<sub>4</sub> curves or  $\gamma$ - and olivine-Ge<sub>3</sub>N<sub>4</sub> curves, which indicates that olivine- $Ge_3N_4$  could be difficult to be observed in experiment. In [13], the bottom point of the olivine-Ge<sub>3</sub>N<sub>4</sub> curve is out of the  $\beta$ -Ge<sub>3</sub>N<sub>4</sub> curve, suggesting the possibility of olivine- $Ge_3N_4$  being detected. The different results arise from the numerical deviation in the total energy calculations, which we believe to be caused by the methodological accuracy. Generally the FLAPW method we employed in this work is taken as a higher accuracy approach in total-energy values due to its all-electron full potential treatment, whereas with respect to this specific system, to clarify the difference definitely needs further calculation by using the pseudopotential method which Dong applied with same internal parameters as ours to see if our FLAPW result could be reproduced; this will be left for the further calculations and studies

The calculated bulk moduli *B* at the equilibrium state are listed in table 3. For  $\gamma$ -Si<sub>3</sub>N<sub>4</sub>, our result of 292.3 GPa is closer to the experiment value 317 GPa with underestimation by 8% than other calculations, while for  $\gamma$ -Ge<sub>3</sub>N<sub>4</sub>, 216.3 GPa underestimates the experimental value by 27%, which is a large deviation than expected, but it is closer to Dong's theoretical value [13]. Our results for olivine phases reveal that olivine-Si<sub>3</sub>N<sub>4</sub>, with 262.9 GPa bulk modulus, is a very hard substance, and also show that for both Si<sub>3</sub>N<sub>4</sub> and Ge<sub>3</sub>N<sub>4</sub>, the spinel phase is a bit stiffer than the olivine phase, which is expected from a slight densification from  $\gamma$  phase to olivine.

#### 4.2. Electronic band structure

The calculated band structures of olivine are shown in figure 3, (a) olivine-Si<sub>3</sub>N<sub>4</sub> and (b) olivine-Ge<sub>3</sub>N<sub>4</sub>, along with the results of (c)  $\gamma$ -Si<sub>3</sub>N<sub>4</sub> and (d)  $\gamma$ -Ge<sub>3</sub>N<sub>4</sub> for completeness. The top of valence band is set to the zero point of the energy in these graphs. The calculated band gaps for the four systems are listed in table 3; the results for the  $\beta$  phases are also included for comparison.

For these four substances, the band structures appear typically to be semiconductor bands with a direct band gap at the  $\Gamma$  point. The band gap values of olivine-Si<sub>3</sub>N<sub>4</sub> and olivine-Ge<sub>3</sub>N<sub>4</sub> are 2.99 and 1.55 eV, and those of  $\gamma$ -Si<sub>3</sub>N<sub>4</sub> and  $\gamma$ -Ge<sub>3</sub>N<sub>4</sub> are 3.21 and 1.90 eV respectively. Both Si<sub>3</sub>N<sub>4</sub> phases have wider band gaps than both phases of Ge<sub>3</sub>N<sub>4</sub>, and the band gaps of  $\gamma$  phases are wider than the ones of olivine phases respectively. Comparing with other reports for spinels, the trends are the same, but our calculations underestimate



Figure 3. Band structures of the four materials: (a) olivine-Si $_3N_4$ , (b) olivine-Ge $_3N_4$ , (c)  $\gamma$ -Si $_3N_4$ , (d)  $\gamma$ -Ge $_3N_4$ .

the energy gap by 7% to 14%. Noticing that the present calculation employs FLAPW, while other calculations adopted orthogonalized linear combination of atomic orbitals (OLCAO) or ultrasoft pseudopotentials, we expect that these differences in values of gap might be owing to the different theoretical treatments of the exchange–correlation potential and basis function set used in expanding the wavefunctions. Since it is well known that LDA (or GGA) calculations

generally underestimate the band gap of semiconductors and insulators, the real band gaps of them must be wider, which suggests that similar to  $\gamma$ -Si<sub>3</sub>N<sub>4</sub>, olivine-Si<sub>3</sub>N<sub>4</sub> is also a potential wide gap semiconductor material.

In fact, the error originates not only from the LDA (or GGA) but also from the DFT itself due to the discontinuity of the exchange–correlation potential. To make it possible to estimate the degree of the calculated error of energy-band gap, we need to compare our results to the experimental ones. However, to our knowledge, the energy gaps of  $\gamma$ -Si<sub>3</sub>N<sub>4</sub> and  $\gamma$ -Ge<sub>3</sub>N<sub>4</sub> have not yet been determined experimentally; let us consider the values of  $\beta$ -phase of Si<sub>3</sub>N<sub>4</sub> and Ge<sub>3</sub>N<sub>4</sub>. As listed additionally in table 3, the calculated energy gaps of  $\beta$ -Si<sub>3</sub>N<sub>4</sub> and  $\beta$ -Ge<sub>3</sub>N<sub>4</sub> are 4.29 eV (indirect), 2.17 eV (direct), while the experimental values are 4.6–5.3, 4.5 eV, respectively. It can be seen that for Si<sub>3</sub>N<sub>4</sub> the error is moderate, about 0.31–1.01 eV, whereas for Ge<sub>3</sub>N<sub>4</sub>, the approximate 2 eV band gap error is about twice that found for Si<sub>3</sub>N<sub>4</sub>, which is the same trend as Dong's [13]. Nevertheless, we also think that tendency showed in the calculated gaps for different phases is informative for further experiments.

The band structures of olivine-Si<sub>3</sub>N<sub>4</sub> and olivine-Ge<sub>3</sub>N<sub>4</sub> look similar; so do the cases of  $\gamma$  phase, since Si and Ge are isoelectronic. The upper and lower valence band widths are nearly identical. According to our analysis, the top of valence bands in both olivine-Si<sub>3</sub>N<sub>4</sub> and olivine-Ge<sub>3</sub>N<sub>4</sub> are dominated mainly by p orbital of N5 and N6. The major difference is in the value of band gap and the states near the conduction band minimum. In olivine-Si<sub>3</sub>N<sub>4</sub> the state at  $\Gamma$  point is dominated mostly by the s and p orbitals of Si2 and the p orbital of N6, while in olivine-Ge<sub>3</sub>N<sub>4</sub>, it is prevailed by the s orbital of Ge2 and Ge1, and the p orbital of N6, N5 and N4. Although an exact discussion of transitions from valence band to conduction band requires calculations of the electronic structures in excited states, as an approximation, the current static electronic band still provides very important information. So it can be approximately identified that the interband transition in olivine-Si<sub>3</sub>N<sub>4</sub> will be from N5 and N6 to Si2 and in olivine-Ge<sub>3</sub>N<sub>4</sub> it will be from N5 and N6 to Ge2, N4 and Ge1, which will lead to the different behaviour in optical properties of the systems. It is also be noticed that within these four crystals, only the minimum of conduction band in  $\gamma$ -Si<sub>3</sub>N<sub>4</sub> is a three-fold degenerate one.

## 4.3. Density of states

The total electronic density of states (DOS), interstitial electronic density of states and sitedecomposed partial density of states (PDOS) of olivine- $Si_3N_4$  and olivine- $Ge_3N_4$  are shown in figure 4 (for three kinds of N atom; since their PDOSs are very similar, only N6 is presented). Like the band structures in figure 3, the DOSs of olivine- $Si_3N_4$  and olivine- $Ge_3N_4$  also look similar on the gross scale. On close inspection, subtle differences can be seen.

In both cases, the total DOSs have a sharp edge near the top of the valence band, which implies that a lot of electrons are distributed there. The valence band is separated into two parts in both: the lower-energy part is dominated by the s orbital of N atoms, the upper part is formed mainly by the p orbital of N atoms with mixing of a portion of s and p orbitals of the Si(Ge) atom; the interaction between these orbitals provides the strong covalent Si(Ge)–N bonding of the system. For the conduction band, a subtle difference in the peaks above the Fermi level between Si<sub>3</sub>N<sub>4</sub> and Ge<sub>3</sub>N<sub>4</sub> can be observed, which can be attributed to the different contributions of atoms in octahedral sites in Si<sub>3</sub>N<sub>4</sub> and Ge<sub>3</sub>N<sub>4</sub>. Observing the PDOS of each atom in both systems, one can see the different electronic contributions for octahedral site atoms (Si1 or Ge1, Si2 or Ge2) and tetrahedral ones (Si3 or Ge3) due to the different local environments.

In olivine-Si<sub>3</sub>N<sub>4</sub>, from the bottom of the conduction band to about 5 eV, s and p orbitals of octahedrally coordinated Si2 prevail mainly, incorporating the p orbitals of N atoms. At about



Figure 4. The total electronic density of states and site-projected partial electronic density of states of (a) olivine- $Si_3N_4$  and (b) olivine- $Ge_3N_4$  at the Si1(Ge1), Si2(Ge2), Si3(Ge3), and N6 site. Fermi energy is at zero point.



Figure 5. Comparison of the charge distribution on (010), (110), (101), (040) planes of olivine- $Si_3N_4$  and olivine- $Ge_3N_4$ .

3.6 eV, there is a peak mainly from s and p orbitals of the Si2 atom. It is noticeable that there exists about a quarter of the total valence electrons in the interstitial region which does not belong to muffin-tin sphere of any atom. This is because the olivine phase structure is not close packed and the interstitial space is rather large. Calculation results show that the electrons in the interstitial space are contributed mostly by the valence electrons of Si atoms.

Olivine-Ge<sub>3</sub>N<sub>4</sub> has a similar situation in valence band as in Si<sub>3</sub>N<sub>4</sub>. However, some different features are observed in Ge<sub>3</sub>N<sub>4</sub> at the bottom of the conduction band. From the bottom of the conduction band to about 2.5 eV, the s orbitals of octahedral Ge2 and tetrahedral Ge3 occupy states together, with interaction with p orbitals of N atoms; this is different feature from the case in Si<sub>3</sub>N<sub>4</sub>, so that in the process of interband transition, electrons will move mainly from the p orbital of the N atom to the octahedrally coordinated Si atom in Si<sub>3</sub>N<sub>4</sub>, but to both octahedral and tetrahedral Ge atoms in Ge<sub>3</sub>N<sub>4</sub>, which induces some differences in optical properties.

## 4.4. Charge-density distributions

The valence charge densities of olivine- $Si_3N_4$  and olivine- $Ge_3N_4$  are shown in figure 5 where four typical planes ((010), (110), (101), (040)) are displayed together.

The different environment of N atoms in  $\gamma$  and olivine structures under pressure is noticed from the bond lengths in table 4. In the  $\gamma$  spinel phase, the N atom is perfectly tetrahedral or octahedral with strong covalent bonding to Si or Ge, whereas in the olivine structure, the atomic volume decreases slightly from that of the spinel phase: the distances of Si or Ge in a tetrahedral site to the N atom are shortened, but those of Si or Ge in an octahedral site to the N atom vary from shorter to longer than the uniform value in spinel. This implies that some ionic bonding character is introduced in the olivine phase. Considering the fact that the olivine

	Olivine-Si <sub>3</sub> N <sub>4</sub>	$\gamma$ -Si <sub>3</sub> N <sub>4</sub>	Olivine-Ge <sub>3</sub> N <sub>4</sub>	γ-Ge <sub>3</sub> N <sub>4</sub>
Bond length (au)				
Octahedral site				
Si1 or Ge1	$3.5448(2 \times N4)$	3.6223	$3.7224(2 \times N4)$	3.8022
	$3.5763(2 \times N5)$		$3.8332(2 \times N5)$	
	$3.6849(2 \times N6)$		$3.9721(2 \times N6)$	
Si2 or Ge2	3.8564(N4)		3.9713(N4)	
	3.6713(N5)		3.8779(N5)	
	$3.2834(2 \times N6)$		$3.7011(2 \times N6)$	
	$4.5035(2 \times N6)$		$4.3731(2 \times N6)$	
Tetrahedral site				
Si3 or Ge3	3.2029(N4)	3.4771	3.3988(N4)	3.6286
	3.2989(N5)		3.5608(N5)	
	$3.1802(2 \times \text{N6})$		$3.4674(2 \times N6)$	

Table 4. Bond lengths in the four materials.

phases have a smaller value of bulk modulus than spinel phases in both  $Si_3N_4$  and  $Ge_3N_4$ , based on our results, this leads to a picture that interaction between the octahedral Si or Ge and N takes more important role in bonding than those of tetrahedral sites.

From figure 5, one can see that in the olivine structure, N is larger than the cations, which is due to a rather large portion of valence electrons of Si or Ge atoms being scattering in the interstitial space. The valence charge distribution of N atoms is non-spherical, which reflects the influences of the asymmetrical surrounding atomic configuration, and some covalent bonding character among them. The Ge atom looks to be a much larger ion than Si since the semi-core-like Ge 3d electrons are not strictly confined to the region near the nucleus. It is also noticed that there is a denser charge distribution between N and Si, indicating a stronger bonding, which might explain the reason why olivine-Si<sub>3</sub>N<sub>4</sub> has a higher bulk modulus than olivine-Ge<sub>3</sub>N<sub>4</sub>.

### 4.5. Optical property

Based on the random phase approximation (RPA) [29], an approximation by many-body perturbation theory to the response of the electron system to a time-dependent electromagnetic perturbation caused by the incoming light, the optical property can be calculated by the real and imaginary parts of the dielectric function  $\varepsilon$ . The imaginary dielectric function  $\varepsilon_2(\omega)$  can be expressed as

$$\varepsilon_{2}(\omega) = \frac{4\pi^{2}e^{2}\hbar}{3m^{2}\omega^{2}} \sum_{ij} \frac{2}{(2\pi)^{3}} \int_{BZ} |M_{ij}(k)|^{2} \delta[\omega_{ij}(k) - \omega] \,\mathrm{d}^{3}k \tag{1}$$

where  $\omega$  is the frequency of incoming light,  $M_{ij}$  is momentum matrix element, and *m* is electron mass. The real dielectric function  $\varepsilon_1(\omega)$  is obtained from  $\varepsilon_2(\omega)$  by the Kramer–Kröning conversion,

$$\varepsilon_1(\omega) = 1 + \frac{2}{\pi} \wp \int_0^\infty \frac{\omega' \varepsilon_2(\omega')}{\omega'^2 - \omega^2} \, \mathrm{d}\omega' \tag{2}$$

where  $\wp$  is the principle value

$$\wp \int_{0}^{\infty} \equiv \lim_{\Delta \to 0} \left( \int_{0}^{\omega - \Delta} + \int_{\omega + \Delta}^{\infty} \right).$$
(3)



**Figure 6.** Dielectric function  $\varepsilon_1(\omega)$ ,  $\varepsilon_2(\omega)$  and energy loss function of  $\gamma$ -, olivine-Si<sub>3</sub>N<sub>4</sub> and Ge<sub>3</sub>N<sub>4</sub>.

Employing the wavefunction of 126 *k*-points in the irreducible BZ from the preceding calculation, the frequency-dependent real and imaginary parts of the dielectric function for photon energies up to 30 eV are estimated. The energy loss function can also be evaluated from the dielectric function. The results for the olivine and spinel phases of Si<sub>3</sub>N<sub>4</sub> and Ge<sub>3</sub>N<sub>4</sub> are shown in figure 6. The solid, dashed and dotted lines represent  $\varepsilon_1(\omega)$ ,  $\varepsilon_2(\omega)$  and the energy loss function, respectively. Because of the anisotropy of the olivine structure, the average of three directions of dielectric function is calculated.

The imaginary dielectric function  $\varepsilon_2(\omega)$  shown in equation (1) represents the optical absorption in these materials. The spectrum of olivine-Si<sub>3</sub>N<sub>4</sub> has a plane absorption peak while olivine-Ge<sub>3</sub>N<sub>4</sub> has a sharp one. The absorption peak values of olivine-phase Si<sub>3</sub>N<sub>4</sub> and Ge<sub>3</sub>N<sub>4</sub> and spinel-phase Si<sub>3</sub>N<sub>4</sub> and Ge<sub>3</sub>N<sub>4</sub> are 7.96, 9.68, 11.1 and 10.9; they are located at photon energies of 9.51, 6.71, 9.73 and 7.03 eV respectively. It is known that the high peak of  $\varepsilon_2(\omega)$  provides fine optical absorption. We noticed that the peaks of  $\varepsilon_2(\omega)$  in these four systems are not as high as those of some well know semiconductors, such as Si and GaAs. Further analysis of the details on the electron interband transformation will provide hints in improving the optical absorption property of the materials.

The static dielectric constant, which is  $\varepsilon_1(0)$  (excluding any contribution from lattice vibrations), is 5.78, 7.48, 5.39 and 7.08 respectively.  $\varepsilon_1(0)$  of both forms of Ge<sub>3</sub>N<sub>4</sub> are larger than those of Si<sub>3</sub>N<sub>4</sub>.

The energy loss functions are also provided in figure 6. The main peak in the energy loss function is defined as the plasma frequency  $\omega_p$ , the frequency of collective oscillation of the valence electrons in the crystal. Those peaks can be identified from the plot:, 27.7, 23.8, 27.2 and 26.1 eV, for olivine-phase Si<sub>3</sub>N<sub>4</sub> and Ge<sub>3</sub>N<sub>4</sub> and spinel-phase Si<sub>3</sub>N<sub>4</sub> and Ge<sub>3</sub>N<sub>4</sub>, respectively.

### 5. Conclusions

A comparative structural and electronic study of a new class of materials, olivine-Si<sub>3</sub>N<sub>4</sub> and olivine-Ge<sub>3</sub>N<sub>4</sub> has been achieved by a first principles approach. The structure optimization provides the detailed information on internal structure parameters for these two olivine crystals. The stability, electronic band structure, equilibrium structural properties, as well the optical property, are evaluated through this work. Our calculation indicates that olivine-Si<sub>3</sub>N<sub>4</sub> might be observed under high pressure and temperature, whereas olivine-Ge<sub>3</sub>N<sub>4</sub> would be difficult to be detected. It is found that there is a favourable direct band gap of 2.99 eV and a large bulk modulus of 262.9 GPa for olivine-Si<sub>3</sub>N<sub>4</sub>; its density of electrons at the top of the valence band is also rather large, which shows the possibility of a high peak of  $\varepsilon_2(\omega)$ . The calculation suggests that olivine-Si<sub>3</sub>N<sub>4</sub> can be a semiconductor with interesting properties and potential technological application.

#### References

- [1] Katz R N 1980 Science 208 841
- [2] Belyi V I et al 1988 Silicon Nitride in Electronics, Material Science Monographs vol 34 (New York: Elsevier)
- [3] Kohastu I and McCauley J W 1974 Mater. Res. Bull. 9 917
- [4] Priest H F, Burns F C, Priest G L and Skaar E C 1973 J. Am. Ceram. Soc. 56 395
- [5] Borgen O and Seip H M 1961 Acta Chem. Scand. 15 1789
- [6] Wild S, Grieveson P and Jack K H 1972 Spec. Ceram. 5 385
- [7] Zerr A, Miehe G, Serghiou G, Schwartz M, Kroke E, Riedel R, Fuess H, Kroll P and Boehler R 1999 Nature 400 340
- [8] Leinenweber K, Keeffe M O', Somayazulu M, Hubert H, McMillan P F and Wolf G H 1999 Chem. Eur. J. 5 3076
- [9] Serghiou G, Miehe G, Tschauner A, Zerr A and Boehler R 1999 J. Chem. Phys. 111 4659
- [10] Mo S-D, Ouyang L, Ching W Y, Tanaka I, Koyama Y and Riedel R 1999 Phys. Rev. Lett. 83 5046
- [11] Sekine T, He H, Kobayashi T, Zhang M and Xu F 2000 Appl. Phys. Lett. 76 3706
- [12] Teter D M and Hemley R J 1996 Science 271 53
- [13] Dong J, Sankey O F, Deb S K, Wolf G and McMillan P F 2000 Phys. Rev. B 61 11979
- [14] Dong J, Deslippe J, Sankey O F, Soignard E and McMillan P F 2003 Phys. Rev. B 67 094104
- [15] Gao F, Xu R and Liu K 2005 Phys. Rev. B 71 052103
- [16] Ching W Y and Rulis P 2006 Phys. Rev. B 73 045202
- [17] Ching W Y, Mo S-D, Ouyang L, Rulis P, Tanaka I and Yoshiya M 2002 J. Am. Ceram. Soc. 85 75
- [18] Jansen J F and Freeman A J 1984 Phys. Rev. B 30 561
- [19] Hahn T 1992 International Tables for Crystallography 3rd revised edn, vol A (Dordrecht: Kluwer-Academic)
- [20] Perdew J P and Wang Y 1992 Phys. Rev. B 45 13244
- [21] Perdew J P, Burke S and Ernzerhof M 1996 Phys. Rev. Lett. 77 3865
- [22] Blaha P, Schwarz K, Madsen G K H, Kvasnicka D and Luitz J 2001 WIEN2k An Augmented Plane Wave + Local Orbitals Program for Calculating Crystal Properties (Karlheinz Schwarz, Techn. Universität Wien, Austria) ISBN 3-9501031-1-2
- [23] Schwarz K, Blaha P and Madsen G K H 2002 Comput. Phys. Commun. 147 71
- [24] Madsen G K H, Blaha P, Schwarz K, Sjöstedt E and Nordström L 2001 Phys. Rev. B 64 195134
- [25] Blochl P E, Jepsen O and Andersen O K 1994 Phys. Rev. B 49 16223
- [26] Fox P A, Hall A D and Schryer N L 1978 The PORT mathematical subroutine library ACM Trans. Math. Softw. 4 104–26
- [27] Ching W Y, Mo S-D and Ouyang L 2001 Phys. Rev. B 63 245110
- [28] Murnaghan F D 1944 Proc. Natl Acad. Sci. USA 30 244
- [29] Hedin L 1965 Phys. Rev. 139 A796
- [30] Weinberg Z A and Pollak R A 1975 Appl. Phys. Lett. 27 254
  Karcher R, Ley L and Johnson R L 1984 Phys. Rev. B 30 1896
  Carson R D and Schnatterly S E 1986 Phys. Rev. B 33 2432
  Iqbal A, Jackson W B, Tsai C C, Allen J W and Bates C W 1987 J. Appl. Phys. 61 2947
- [31] Chambouleyron I and Zanatta A R 1998 J. Appl. Phys. 84 1